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On the risk situation of financial conglomerates: does diversification matter?

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Abstract In general, conglomeration leads to diversification of risk (the diversification benefit) and a decrease in shareholder value (the conglomerate discount). Diversification benefits in financial conglomerates are typically derived without explicitly accounting for reduced shareholder value. However, a comprehensive analysis requires competitive conditions within the conglomerate, i.e., shareholders and debt holders should receive risk-adequate returns on their investment. In this paper, we contribute to the literature on this topic by comparing the diversification effect in conglomerates with and without accounting for altered shareholder value. We derive results for a holding company, a parent-subsidiary structure, and an integrated model. In addition, we consider different types of capital and risk transfer instruments in the parent-subsidiary model, including intragroup retrocession and guarantees. We conclude that under competitive conditions, diversification does not matter to the extent frequently emphasized in the literature. The analysis contributes to the ongoing discussion on group solvency regulation and enterprise risk management, which is of relevance to insurance groups and other financial conglomerates.

Keywords Financial conglomerate · Diversification · Risk-neutral valuation · Conglomerate discount

JEL Classification G13 · G20 · G28 · G32

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1 Introduction

In an environment of increasingly frequent consolidation activity, the advantages and risks of corporate diversification are of great interest to regulatory authorities and financial group management.[1] In general, conglomeration leads to diversification of risk (the diversification benefit) but also to a decrease in shareholder value (the conglomerate discount). These two effects have not been analyzed simultaneously in the literature to date, in that diversification benefits are typically calculated using the concept of economic solvency capital without accounting for the reduction in shareholder value, even though a comprehensive analysis requires a competitive situation in financial conglomerates (i.e., shareholders and debt holders receive risk-adequate returns on their investments). In this paper, we extend the literature and compare diversification benefits and insolvency risks in groups with and without accounting for the reduced shareholder value using risk-neutral valuation. To attain a more profound understanding of the effects of diversification, we derive results for a holding company, a parent-subsidiary structure, and an integrated model. In addition, we consider different types of capital and risk transfer instruments (CRTIs)—which are legally enforceable agreements between two entities of the group—in the parent-subsidiary model, including intragroup retrocession and guarantees.

The extent of diversification effects and conglomerate discount depends on the specific organizational form and is contingent on capital and risk transfer instruments. Different legal structures of conglomerates, relevant risks, and benefits are discussed in Diereck (2004); in addition, Basu (2010) analyzes firms' diversification and refocusing strategies. The holding company model is representative of the stand-alone case, as the entities fail independently and, therefore, no portfolio effects arise if no transfer of assets takes place. Integrated financial conglomerates have a single, consolidated balance sheet and must satisfy a single solvency capital requirement. Therefore, they benefit fully from diversification effects, but also face risk concentration (see Allen and Jagtiani 2000; Mälkönen 2004; Gatzert et al. 2008). In a parent-subsidiary structure, single entities can generally default without causing others to do the same. However, the subsidiary's market value is an asset for the parent. In this setting, two concepts can be distinguished with respect to the diversification of risks. First, group-level diversification occurs if the risks of different legal entities in a group are not fully correlated. Second, down-streaming of diversification occurs if CRTIs are in place.

The diversification benefit is typically measured based on the conglomerate's economic capital relative to the sum of stand-alone economic capital. In the context of regulation, Keller (2007) and Luder (2007) discuss the group-level Swiss Solvency Test and how CRTIs are accounted for when measuring the solvency capital requirements of insurance groups in a parent-subsidiary structure. In a similar setting,

[1] In this paper, we use the terms "financial group" and "financial conglomerate" interchangeably. A definition of financial conglomerates is given in Diereck (2004, p. 10): "In the most general sense, a financial conglomerate is a group of entities whose primary business is financial and whose regulated entities engage to a significant extent in at least two of the activities of banking, insurance and securities. According to this definition, bancassurance groups would qualify as financial conglomerate, but so would groups combining insurance and securities or banking and securities."

Filipovic and Kupper (2007a, 2007b) derive optimal CRTIs that minimize the difference between available and required capital in an insurance group for convex risk measures, thus focusing on the group perspective. Others take a different approach and use principal-agent models. Freixas et al. (2007), e.g., compare the risk-taking incentives of stand-alone firms, holding company conglomerates, and integrated conglomerates, and show that diversification within integrated models can increase risk-taking incentives, and thus lower social welfare relative to the stand-alone case. Loranth and Morrison (2007) examine the effect of a multinational bank's liability structure, showing that diversification is unattractive in the presence of fixed bank capital requirements. Kahn and Winton (2004) derive an optimal subsidiary structure for financial institutions given moral hazard between group and subsidiary management. Devos et al. (2009) empirically estimate average synergy gains of mergers to 10.03% of the combined equity, which is decomposed into operating and financial synergies. The authors find that mergers generate value chiefly by improving resource allocation rather than by reducing tax payments or increasing market power.

In respect to the conglomerate discount, Berger and Ofek (1995) empirically show for the US market that there was a reduction in firm value of between 13% and 15% between 1986 and 1991, which they attribute, in part, to overestimation and cross-subsidization. Focusing on banks, Laeven and Levine (2007) also observe a conglomerate discount and stress agency problems as a possible cause. In agency theory, the conglomerate discount on firm value has been explained by asymmetric information distribution, which implies that managers do not necessarily act in the best interests of their equity holders, but instead to increase their own personal wealth (see Amihud and Lev 1981; Jensen 1986, 1993; Jensen and Murphy 1990). Schmid and Walter (2009) investigate the conglomerate discount in a sample of US financial intermediaries covering the whole financial industry, including commercial banking, investment banking, and insurance, and analyze the valuation effect of specific combinations of financial activity. They find that these firms trade at a substantial and consistent discount, which is due to diversification and is not based on the strategy that troubled firms diversify into other more promising areas. Van Lelyveld and Knot (2009) investigate the conglomerate discount in a sample of European bank-insurance conglomerates and find that the diversification discount is not universal, but varies significantly depending on size, complexity, and risk, among other factors. Based on financial theory, Ammann and Verhofen (2006) explain and quantify the conglomerate discount using Merton's structural model and attribute the discount to the equity holders' limited liability. Mansi and Reeb (2002) first point out the risk effects of conglomeration implying a value transfer from shareholders to debtholders, referring to the classical conflict of interest between these two stakeholder groups regarding the risk profile of the firm, as risk reduction implies an increase in the market value of debt. They conduct an empirical analysis to confirm their prediction and show that the book value of debt bias implies a considerable underestimation of diversified firm value and that, when using market values of debt, diversification has no impact on the overall firm value.

There is another research stream that questions the existence of a conglomerate discount due to diversification effects and argues that the discount is due to endogenous reasons of diversification because weaker firms are the ones that choose to diversify (Villalonga 2004; Campa and Kedia 2002). However, a majority of articles find significant empirical evidence of a conglomerate discount. More recent studies investigate the question of whether the conglomerate discount is really due to a reduction in firm risk and thus a value transfer from shareholders to debtholders as pointed out in Mansi and Reeb (2002). Grass (2009), for instance, calculates the expected conglomerate discount resulting from the risk-reducing effect of diversification in a contingent claims framework and finds that the discount attributable to diversification amounts to only 0.9% for the mean multisegment firm in a US sample. Glaser and Mueller (2010) use alternative specifications of the Merton (1974) bond pricing model to estimate the market value of debt in a sample of nonfinancial German firms and find the conglomerate discount to be reduced but, in contrast to Mansi and Reeb (2002), that it does not entirely disappear when using market values of debt instead of book values. They conclude that the book value bias of debt is only one explanation for the conglomerate discount. Ammann et al. (2009) also find that the effect from using the market value of debt based on the Merton (1974) model and an alternative approach based on Damodaran (2005) is very limited compared to using the book value of debt. Furthermore, they show that the debt value is affected by organizational structure only when the degree of diversification changes and that the conglomerate discount is still significant even when accounting for endogeneity of the diversification decision.

Thus, the cited literature either quantifies the conglomerate discount or measures diversification benefits with respect to solvency capital, but nowhere in this body of work are the two concepts combined. Furthermore, for the most part, parent-subsidiary models and the effect of CRTIs are not considered. When comparing diversification effects and insolvency risk within different conglomerate structures, the corresponding fair capital structure differs. In particular, we expect stakeholders to adjust their capital structure in order to achieve risk-adequate returns whenever the group structure changes. This is an important aspect that has not received attention in the literature on diversification to date, even though it has major implications for group management decisions and solvency regulation. Our aim is to fill this gap and provide a better understanding of a financial conglomerate's risk situation by conducting an analysis that looks at conglomerate discount and diversification effects simultaneously.

We first provide a model framework for the different financial group structures and then proceed as follows. For two entities, we first keep the capital structure fixed and study diversification (as done in Keller 2007 and Luder 2007) and insolvency risk, thereby adding a new perspective by comparing results for different organizational forms (parent-subsidiary model, holding company, and integrated conglomerate). Furthermore, we account for CRTIs and include a guarantee from parent to subsidiary and quota share retrocession, i.e., the parent pays a share of the subsidiary's liabilities (e.g., Filipovic and Kupper 2007a, 2007b). Second, we adjust the equity capital for each type of conglomerate using a financial approach for modeling the conglomerate discount—similar to the one used in Ammann and Verhofen (2006) for the case of an integrated conglomerate—and then conduct the same analysis. This ensures a competitive situation for each type of conglomerate, i.e., the value of the equity holders' payoff equals their initial contribution. Thus, we compare the diversification benefit with and without accounting for the conglomerate discount, which varies depending on the type of financial conglomerate, and tends toward zero with increasing dependence between the two firms. We further illustrate our theoretical considerations and derived results using numerical examples based on Monte Carlo simulation. The conglomerate discount is thus quantified by employing a financial option-based approach; the diversification benefit is calculated using the tail value at risk and the shortfall probability.

In this analysis, we take the group-level perspective. i.e., diversification benefit and joint default probabilities, as well as perspective of the individual institutions, i.e., solvency capital and individual shortfall risk, to provide a detailed picture of the altered group situation. We conclude that, for the conglomerates considered here, when diversification effects are studied under competitive conditions, diversification related to risk reduction does not matter to the extent frequently emphasized in the literature. More precisely, we argue that competitive conditions should in general lead to a situation in which shareholders and debtholders receive risk-adequate returns on their investment and hence possible reactions from the two major stakeholder groups should be taken into account when studying diversification effects. Under competitive conditions, advantages from diversification within a financial conglomerate are not as great as is frequently emphasized in the literature. We believe this aspect to be important not only from the viewpoint of financial conglomerate management, but also in respect to current considerations of adequate risk-based capital standards. Hence, our analysis contributes to the ongoing discussion on group solvency regulation (Swiss Solvency Test, Solvency II) as well as that regarding enterprise risk management.

The remainder of the paper is organized as follows. Section 2 introduces the model of the stand-alone institutions and the corresponding fair valuation, solvency capital, and shortfall-risk calculations. Different corporate structures of financial conglomerates and capital and risk transfer instruments are discussed in Sect. 3. Section 4 analyzes the measurement of diversification benefits and conglomerate discount in the considered group structures. To illustrate our theoretical results, a simulation analysis of diversification benefits with and without accounting for the conglomerate discount is conducted in Sect. 5. Section 6 relates our theoretical and numerical findings to other recent empirical research and Sect. 7 summarizes the results.

2 Modeling stand-alone institutions

We consider a firm with a market value of liabilities L_t and a market value of assets A_t at $t = 0, 1$. In this one-period setting, debtholders and equity holders make initial payments of D_0 and E_0, respectively. The sum of the initial contributions $A_0 = D_0 + E_0$ is invested in the capital market. At time $t = 1$, debtholders receive the value of the liabilities, and equity holders receive the remainder of the market value of the assets. If the company is not able to cover the liabilities, the total value of the assets is distributed to the debtholders and the equity holders receive nothing. The debt holders' payoff D_1 is thus expressed by:

$$D_1 = L_1 - \max(L_1 - A_1, 0),$$

where the second term represents the payoff of the default put option (*DPO*) (see Doherty and Garven 1986). The payoff to equity holders, E_1, is accordingly given by the remainder:

$$E_1 = A_1 - D_1 = \max(A_1 - L_1, 0).$$

To model the development of assets and liabilities, we use a geometric Brownian motion (see, e.g., Gatzert and Schmeiser 2008a, 2008b in regard to insurance companies, and Ammann and Verhofen 2006 in respect to banks). Under the real-world measure $\mathbb{P}$, the stochastic processes are described by:

$$dA(t) = \mu_A A(t)\,dt + \sigma_A A(t)\,dW_A^{\mathbb{P}}(t),$$
$$dL(t) = \mu_L L(t)\,dt + \sigma_L L(t)\,dW_L^{\mathbb{P}}(t),$$

with μ and σ denoting the drift and volatility (assumed to be constant over time) of the stochastic processes. $W_A^{\mathbb{P}}$ and $W_L^{\mathbb{P}}$ are standard $\mathbb{P}$-Brownian motions with a correlation of coefficient ρ, i.e., $dW_A\,dW_L = \rho(A, L)\,dt$. Given values for $A(0) = A_0$ and $L(0) = L_0$, the solutions of the stochastic differential equations above are given by (see, e.g., Björk 2004):

$$A(t) = A_0 \cdot \exp\big((\mu_A - \sigma_A^2/2)t + \sigma_A W_A^{\mathbb{P}}(t)\big),$$
$$L(t) = L_0 \cdot \exp\big((\mu_L - \sigma_L^2/2)t + \sigma_L W_L^{\mathbb{P}}(t)\big).$$

Changing the real-world measure $\mathbb{P}$ to the equivalent risk-neutral martingale measure $\mathbb{Q}$ leads to the constant riskless rate of return r as the drift of the processes.

Risk-neutral valuation Valuation of the claims is conducted using risk-neutral valuation (also referred to as "fair valuation"). From the debtholders' perspective, a fair price for their claims (subject to default risk) satisfies the following condition (see, e.g., Doherty and Garven 1986), which can be solved in closed form as the claims represent European options with a stochastic strike price (Margrabe 1978; Fischer 1978):

$$\begin{aligned} D_0 &= E^{\mathbb{Q}}\big(\exp(-r) \cdot L_1\big) - E^{\mathbb{Q}}\big(\exp(-r) \cdot \max(L_1 - A_1, 0)\big) = L_0 - \Pi_0^{DPO} \\ &= L_0 - \big(L_0 N(d_1^D) - A_0 N(d_2^D)\big) \end{aligned} \tag{1}$$

with

$$d_1^D = \frac{\ln(L_0/A_0) + \hat{\sigma}^2/2}{\hat{\sigma}}, \quad d_2^D = d_1^D - \hat{\sigma}, \quad \hat{\sigma} = \sqrt{\sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A\sigma_L}.$$

Hence, the debtholders' initial payment must equal the nominal value of liabilities less the value of the *DPO* at $t = 0$. Considering the risk parameter $\hat{\sigma} = \sqrt{\sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A\sigma_L}$, insolvency risk is, ceteris paribus, reduced for a positive correlation between assets and liabilities. Given a fixed safety level measured with the *DPO* value Π_0^{DPO} and the value for the nominal liabilities L_0, the contribution of the debt holders D_0 is fixed (see, e.g., Gatzert and Schmeiser 2008a). Due to no arbitrage, (1) also implies:

$$E_0 = E^{\mathbb{Q}}\big(\exp(-r)\cdot E_1\big) = E^{\mathbb{Q}}\big(\exp(-r)\cdot \max(A_1 - L_1, 0)\big)$$
$$= A_0 N\big(d_1^E\big) - L_0 N\big(d_2^E\big)$$

where

$$d_1^E = \frac{\ln(A_0/L_0) + \hat{\sigma}^2/2}{\hat{\sigma}}, \quad d_2^E = d_1^E - \hat{\sigma}, \quad \hat{\sigma} = \sqrt{\sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A\sigma_L}.$$

Thus, the payment by the equity holders equals the value of their payoff at time $t = 1$.

Solvency capital Based on a given capital structure (E_0, D_0), available and necessary economic capital can be derived that ensures the firm remains solvent. In banking and insurance regulation (see, e.g., Basel II, Solvency II, Swiss Solvency Test), the firm's available economic capital is often called risk-bearing or risk-based capital (*RBC*), which is defined as the market value of assets less the market value of liabilities at time t (see, e.g., Keller 2007):

$$RBC_t = A_t - L_t.$$

The solvency (or target) capital (*SC*) required is the amount of capital needed at $t = 0$ to meet future obligations over a fixed time horizon for a required confidence level α. In general, regulators require that the firm's solvency capital will not exceed the risk-bearing capital in $t = 0$:

$$RBC_0 \geq SC.$$

The amount of necessary economic capital depends on the risk measure chosen. In the following, we use the tail value at risk (*TVaR*) for a given confidence level α, which is more restrictive than the value at risk.[2] Hence, *SC* can be derived by

$$SC = TVaR_\alpha = -E^{\mathbb{P}}\big(\exp(-r)\cdot RBC_1 \big| \exp(-r)\cdot RBC_1 \leq VaR_\alpha\big) + RBC_0, \qquad (2)$$

where VaR_α is the value at risk for a confidence level α given by the quantile of the distribution $F^{-1}(\alpha) = \inf\{x : F(x) \geq \alpha\}$.[3] Thus, to satisfy the regulatory requirement $RBC_0 \geq SC$, one can check whether

$$E^{\mathbb{P}}\big(\exp(-r)\cdot RBC_1 \big| \exp(-r)\cdot RBC_1 \leq VaR_\alpha\big) \geq 0.$$

The amount of solvency capital further depends on the input parameters and the underlying stochastic model.

[2] The *TVaR* is implemented in insurance regulation in Switzerland, while the value at risk is the required risk measure in the European banking regulation (Basel II) and also planned for the European insurance regulatory framework Solvency II. For a discussion of value at risk and tail value at risk, see, e.g., Chen and Lin (Chen and Lin 2006, p. 381 ff).

[3] Equation (2) is equivalent to: $SC = TVaR_\alpha = -E^{\mathbb{P}}(\exp(-r)RBC_1 - RBC_0 | \exp(-r)RBC_1 - RBC_0 \leq VaR_\alpha)$, which corresponds to the change in *RBC* within one year, where RBC_1 is discounted with the riskless interest rate r.

Shortfall risk A legal entity's shortfall probabilities are calculated by

$$SP = \mathbb{P}(RBC_1 < 0) = \mathbb{P}(A_1 < L_1).$$

Shortfall is thus defined as the event where available economic capital falls below zero, i.e., the firm is insolvent.[4]

3 Corporate structures of financial conglomerates

The previous section set out fair valuation and solvency capital calculations for stand-alone firms. These calculations can be substantially different for financial conglomerates where the type of conglomerate structure plays an important role in risk and capital requirements. A detailed discussion of conglomeration and regulatory issues involved in the supervision of financial conglomerates in the European Union can be found in Diereck (2004). In the following, we present three types of conglomerates that differ with respect to ownership: the holding company model, the parent-subsidiary model, and the integrated model. The financial conglomerate we consider consists of two legal entities, (P) and (S).

Holding company In the holding company model, an umbrella corporation owns the two entities. Operationally, the firms are separate and also must be separately capitalized as they have no access to each others' cash flows. In an umbrella corporation, certain tasks, such as risk management, capital raising and allocation, and IT, are typically centralized (Diereck 2004). Thus, without considering synergy effects, in essence, the holding company model is similar to the case of two stand-alone firms since the holding company does not benefit from portfolio effects. In the following numerical analysis, we compare the uncorrelated case to the case where there is a highly positive correlation coefficient regarding the cash flows of the two legal entities, which may be due to a high degree of centralization in the group.

Parent-subsidiary model In the parent-subsidiary model, the parent owns the subsidiary but the two companies remain legally and operationally separate. As in the holding company model, the firms are separately capitalized, and the parent company is not obliged to cover the subsidiary's liabilities in the absence of legally binding capital and risk transfer instruments (CRTIs). On the other hand, the parent has direct access to the subsidiary's profits. Thus, the market value of the subsidiary is an asset to the parent. In the analysis, we assume that the subsidiary will continue in business after $t = 1$. Thus, the firm must meet at least certain minimum capital requirements (MCR^S), and the available capital at $t = 1$ must be $\min(A_1^S - L_1^S, MCR^S)$. Therefore,

[4] Even in case of default, banks and insurance companies have often continued in business by receiving aid from the government. There may also be incentive problems in the event a company faces insolvency. Wilson and Wu (2010), e.g., show that if a bank is facing insolvency, it will be tempted to reject good loans and accept bad loans so as to shift risk to its creditors.

the subsidiary's market value $(A_1^S - L_1^S)$ will not be fully extracted and the maximum transferable value to the parent is given by

$$\max\left(A_1^S - L_1^S - MCR^S, 0\right).$$

It is assumed that the parent can sell the subsidiary for this value. Limiting the market value to *MCR* can be considered as a form of regulatory costs (see, e.g., Filipovic and Kupper 2007b).

In the parent-subsidiary model, we further integrate capital and risk transfer instruments (CRTIs). CRTIs are legally enforceable contractual capital and risk transfer instruments (e.g., FOPI 2006, p. 4), such as dividends, reinsurance agreements, intra-group retrocession, securitization of future cash flows, guarantees, and other contingent capital solutions. However, a parent can offer guarantees only when its financial situation is appropriate to ensure the guarantees. These instruments serve to reduce the subsidiary's solvency capital requirements. When the financial situation is good, capital transfers may also include transfers that are not legally binding. In a situation of financial distress, only legal, contractual agreements can be enforced. The economic (available, risk-bearing) capital of the parent company is thus also affected by the subsidiary's liabilities when CRTIs are in place.

In this analysis, we consider two types of CRTIs: a guarantee from parent to subsidiary and a quota share retrocession. Under the guarantee, the parent company covers the shortfall $DPO^S = \max(L_1^S - A_1^S, 0)$ of the subsidiary in $t = 1$, but only to the extent that its own available capital at time $t = 1$ is at least above the minimum capital necessary for it to continue its own business, i.e., $\min(A_1^P - L_1^P, MCR^P)$. Therefore, the transfer T to the subsidiary is limited to $\max(A_1^P - L_1^P - MCR^P, 0)$. Hence, if the parent offers the subsidiary a guarantee, liability T^G with

$$T^G = \min\left(DPO^S, \max\left(A_1^P - L_1^P - MCR^P, 0\right)\right)$$

is down-streamed as equity to the subsidiary. The other CRTI under consideration is quota share retrocession, where the parent promises to pay a share β of the subsidiary's liabilities:[5]

$$T^R = \min\left(\beta \cdot L_1^S, \max\left(A_1^P - L_1^P - MCR^P, 0\right)\right).$$

Integrated model An integrated conglomerate has one consolidated balance sheet and, in principle, capital is fully fungible between the different entities. In this model, the conglomerate benefits from diversification since losses from failing projects can be offset by returns from successful projects. This situation can lead to increased risk-taking behavior by the entities, i.e., moral hazard due to a "too-big-to-fail" attitude (see, e.g., Diereck 2004). In the European Union, e.g., insurance companies and banks are prohibited from forming this type of conglomerate.

[5] This instrument is also considered by Filipovic and Kupper (2007b) in the context of insurance groups.

4 Diversification benefit and conglomerate discount

On a stand-alone basis, both legal entities in the financial conglomerate, (P) and (S), can be treated as described in Sect. 2. However, if the two firms form a financial conglomerate, these calculations will generally be different due to the ownership relations, as shown in the previous section. Thus, following a general discussion, the three types of group structure will be individually analyzed and discussed with respect to diversification effects and conglomerate discount.

4.1 Diversification benefits in financial conglomerates

We measure diversification effects in a financial group in two ways. First, we compare the shortfall risk of firms at the group level. From the group's perspective, the joint default probabilities of exactly one (P_1) or both entities (P_2) are given by

$$P_1 = \mathbb{P}\big(RBC_1^S < 0, RBC_1^P \geq 0\big) + \mathbb{P}\big(RBC_1^S \geq 0, RBC_1^P < 0\big),$$
$$P_2 = \mathbb{P}\big(RBC_1^S < 0, RBC_1^P < 0\big).$$

Second, the diversification effect in a financial conglomerate is also typically measured based on solvency capital requirements (see, e.g., Filipovic and Kupper 2007a). The relative diversification benefit is given by the sum of capital requirements when taking into account the conglomerate structure, divided by the sum of stand-alone (solo) capital requirements:

$$d^{\text{group}} = 1 - \frac{SC^{P,\text{group}} + SC^{S,\text{group}}}{SC^{P,\text{solo}} + SC^{S,\text{solo}}}.$$

The less solvency capital the group is required to hold, the higher the coefficient d, and thus the higher the conglomerate's degree of diversification. Since the holding company model corresponds to the stand-alone case, no diversification benefits can occur as we do not include a transfer of assets between different legal entities in the model. In general, asset transfers are likely to occur for reputational reasons, for instance. However, in our analysis, we consider only legally binding capital and risk transfers between legal entities when calculating the diversification benefit and conglomerate discount. In the event of further asset transfers, these effects will be enhanced.

Two types of diversification can be distinguished in a parent-subsidiary structure. First, group-level diversification occurs if the cash flows of legal entities in the conglomerate are not fully correlated. In particular, nonperfectly correlated assets and liabilities of parent and subsidiary are beneficial for the parent company in terms of risk reduction, while the subsidiary neither profits nor suffers disadvantages from the ownership relation. Second, down-streaming of diversification occurs when legally binding transfer of losses contracts are in place, which are beneficial for the subsidiary. If no CRTIs are implemented, no contagion effects can occur, and only group-level diversification can arise. In the following, we compare these two cases.

In a fair situation, the subsidiary's debtholders pay a fair premium for the guarantee, which is transferred to the parent company. The guarantee leads to an increase in available economic capital at $t = 1$ for the subsidiary, and to a decrease in same for the parent. It is assumed that the subsidiary's available economic capital at $t = 0$ remains unchanged (and hence equals the solo case), and so the solvency capital requirements remain the same. At time $t = 1$, the available capital is decreased by the parent's participation and increased by the CRTI transfer (denoted by T) from the parent to the subsidiary. Thus, one obtains:

$$RBC_0^S = A_0^S - L_0^S, \qquad RBC_1^S = \min\bigl(A_1^S - L_1^S, MCR^S\bigr) + T.$$

Analogously, the risk-bearing capital of the parent company at $t = 0$ and $t = 1$ is given by

$$RBC_0^P = A_0^P - L_0^P, \qquad RBC_1^P = A_1^P - L_1^P + \max\bigl(A_1^S - L_1^S - MCR^S, 0\bigr) - T.$$

The formulas for the risk-bearing capital show that the CRTI payment will take place only if the parent's financial situation permits it, i.e., after all its own debt is paid. This means that the parent's shortfall risk will not be negatively affected and remains unchanged. Thus, the parent's debtholders will not be worse off if CRTIs are in place. On the other hand, the subsidiary's debtholders will benefit from CRTIs due to a reduction in shortfall risk and reduced solvency capital requirements. In addition, double gearing of capital is avoided since the value of the subsidiary is split in two parts:

$$A_1^S - L_1^S = \max\bigl(A_1^S - L_1^S - MCR^S, 0\bigr) + \min\bigl(A_1^S - L_1^S, MCR^S\bigr).$$

In the integrated model, risk-bearing capital is determined by the difference between the sum of assets and the sum of liabilities of the group's entities:

$$RBC_0^{\text{int}} = A_0^P + A_0^S - L_0^S - L_0^P,$$
$$RBC_1^{\text{int}} = A_1^P + A_1^S - L_1^S - L_1^P,$$

where full fungibility of capital is assumed. Joint shortfall is not defined in the sense described above, but coincides with the individual shortfall probabilities:

$$SP = P_2 = \mathbb{P}\bigl(RBC_1^{\text{int}} < 0\bigr).$$

In this setting, diversification benefits originate as assets, and liabilities are not fully correlated. Table 1 summarizes the risk-bearing capital at Time 1 for the different conglomerate structures used in the following numerical examples.

4.2 Measuring the conglomerate discount

The conglomerate discount is identified by determining the fair capital structure after group building using risk-neutral valuation and by comparing this value to the stand-alone case. In particular, a diversification of risks will imply a reduction in shareholder value (conglomerate discount). Hence, by deriving the fair capital structure in a financial group, we are able to explicitly quantify the conglomerate discount.

Table 1 Risk-bearing capital at $t = 0$ and $t = 1$ for different conglomerate structures

	$t = 0$		$t = 1$	
	RBC_0^P	RBC_0^S	RBC_1^P	RBC_1^S
Holding	$= A_0^P - L_0^P$	$= A_0^S - L_0^S$	$= A_1^P - L_1^P$	$= A_1^S - L_1^S$
Parent-subsidiary	$= A_0^P - L_0^P$	$= A_0^S - L_0^S$	$= A_1^P - L_1^P + \max(A_1^S - L_1^S - MCR^S, 0)$	$= \min(A_1^S - L_1^S, MCR^S)$
Parent-subsidiary with guarantee	$= A_0^P - L_0^P$	$= A_0^S - L_0^S$	$= A_1^P - L_1^P + \max(A_1^S - L_1^S - MCR^S, 0) - T^G$	$= \min(A_1^S - L_1^S, MCR^S) + T^G$
Parent-subsidiary with retrocession	$= A_0^P - L_0^P$	$= A_0^S - L_0^S$	$= A_1^P - L_1^P + \max(A_1^S - L_1^S - MCR^S, 0) - T^R$	$= \min(A_1^S - L_1^S, MCR^S) + T^R$
Integrated conglomerate	$= A_0^P + A_0^S - L_0^S - L_0^P$	–	$= A_1^P + A_1^S - L_1^S - L_1^P$	–

For the holding company model, there is no conglomerate discount as there is no diversification of risks due to no asset transfers. The procedure for determining the conglomerate discount corresponds to the financial approach taken in Ammann and Verhofen (2006), where the focus is on the case of an integrated conglomerate.[6]

In the parent-subsidiary model, one needs to distinguish between fair valuation (leading to a fair capital structure that accounts for the conglomerate discount) and solvency assessment (actual shortfall risk, solvency capital). In the fair valuation process, we assume that the subsidiary separately pays a fair price for any CRTIs, and thus they are not part of the fair initial equity that ensures the preset safety level without the CRTI. Furthermore, the ownership relation (the parent can sell the subsidiary for its market value) has no influence on the subsidiary's debtholders. Hence, the debtholders require the same amount of equity capital in the company as would be the case without CRTI structure. Therefore, the subsidiary's initial situation in the CRTI model is identical to the stand-alone case and no conglomerate discount is present in the case of downstreaming diversification.

The situation is different for the parent company, as its debtholders profit from the possibility of selling the subsidiary at its market value due to the reduction in the *DPO* payoff to

$$DPO_1^P = \max\left(L_1^P - A_1^P - \max\left(A_1^S - L_1^S - MCR^S, 0\right), 0\right), \tag{3}$$

[6] Similar considerations for risk reduction hold in the case of general asset or stock portfolios; see, e.g., Gruber and Elton (1991) and Isakov and Barras (2003).

and thus the debtholders' payoff at $t = 1$ is

$$D_1^P = L_1^P - DPO_1^P.$$

Given the same safety level *DPO* and same nominal value of liabilities (such that the parent's debtholders pay the same amount with and without participation), initial equity capital, in general, can be reduced, thus revealing the existence of a conglomerate discount for the parent, which originates from the ownership relation. To find the new fair initial equity E_0^P, (1) can be solved for E_0^P, which also satisfies the following equation:

$$\begin{aligned} E_0^P &= \Pi_0^S = E^{\mathbb{Q}}\left(\exp(-r) \cdot E_1^P\right) \\ &= E^{\mathbb{Q}}\left(e^{-r} \cdot \max\left(A_1^P - L_1^P + \max\left(A_1^S - L_1^S - MCR^S, 0\right), 0\right)\right). \end{aligned}$$

Hence, the conglomerate discount effect generally implies a reduction in the value of the equity holders' payoff through the participation in the subsidiary.

The conglomerate discount is also distinct in the case of an integrated conglomerate. To allow comparison between the different conglomerate structures, the debtholders of the conglomerate pay the same initial amount as in the stand-alone case. One fair equity-premium combination is then derived by adjusting the equity capital of the subsidiary E_0^S only, leaving everything else as in the stand-alone case:

$$D_0^P + D_0^S = L_0^P + L_0^S - E^{\mathbb{Q}}\left(\exp(-r) \cdot \max\left(L_1^P + L_1^P - A_1^P - A_1^S, 0\right)\right).$$

Since the sum of four geometric Brownian motions is no geometric Brownian motion and also cannot be interpreted as a European option with stochastic strike price as in the previous section, in general no closed-form solution can be derived. Since the debtholders pay the same amount and have the same claims cost distribution in $t = 1$ (and the same nominal value of liabilities $L_0^{P,S}$), to ensure a fair situation, the *DPO* value in the integrated conglomerate must equal the sum of stand-alone *DPO* values:

$$\Pi_0^{DPO,\text{int}} = E^{\mathbb{Q}}\left(e^{-r} \max\left(L_1^P + L_1^S - A_1^P - A_1^S, 0\right)\right) = \Pi_0^{DPO,S} + \Pi_0^{DPO,P}.$$

Furthermore, the fact that

$$\max\left(L_1^P + L_1^S - A_1^P - A_1^S, 0\right) \le \max\left(L_1^P - A_1^P, 0\right) + \max\left(L_1^S - A_1^S, 0\right)$$

implies that, in general, less equity capital is necessary to meet the safety level $\Pi_0^{DPO,\text{int}}$.[7] Therefore, the fair capital structure will also imply a reduction in shareholder value. Since shareholders require risk-adequate returns on their investment, equity capital will be reduced in a financial group (without considering other effects). This conglomerate discount will be higher with a decreasing correlation coefficient between the companies' cash flows.

[7] Ammann and Verhofen (2006) take a similar approach in the integrated case and attribute the conglomerate discount to the limited liability of equity holders. They conduct an analysis of the conglomerate discount under different distributional assumptions and different numbers of business lines in the case of an integrated conglomerate.

As a consequence, the conglomerate discount has a substantial effect on the diversification benefits, since the reduction in equity capital, ceteris paribus, implies a reduced diversification effect regarding shortfall probability and solvency capital. This is evident in the calculation of the solvency capital in (2), where a reduction in equity capital leads to an increase in solvency capital requirements. Similar results hold for the calculation of shortfall risk. For firms that face a choice between whether to merge or type of ownership relation to take, it is vital to account for the conglomerate discount upfront, before estimating future diversification benefits.

5 Numerical analysis

In this section, we provide some numerical illustrations of the theoretical considerations and derived results from the previous sections that are intended to achieve further insight into the central effects of diversification benefits and conglomerate discounts. To this end, the holding company is compared to both the parent-subsidiary model without CRTIs and to the integrated model. In addition, we conduct a deeper analysis of the impact of introducing CRTIs into the parent-subsidiary model, i.e., guarantees and retrocession. The holding company model essentially corresponds to the stand-alone case of the two firms.

Input parameters As input parameters we set the confidence level for the *TVaR* to $\alpha = 1\%$ (as required, e.g., by the Swiss Solvency Test). We consider two firms, (P) and (S), which have the same safety level, are the same size, and have the same asset and liability structure.[8] The *DPO* value of both firms is fixed at $\Pi_0^{DPO} = 0.1$ and the nominal value of liabilities is given by $L_0^S = L_0^P = 100$. Therefore, according to the fairness condition in (1), the debtholders' contribution for both firms is given by

$$D_0 = L_0 - \Pi_0^{DPO} = 100 - 0.1 = 99.9.$$

Drift and standard deviation of the assets and liabilities of (P) and (S) are set to $\mu_A = 0.09$, $\sigma_A = 0.10$ (for the assets) and $\mu_L = 0.01$, $\sigma_L = 0.10$ (for the liabilities). The coefficients of correlation between assets and liabilities of the subsidiary and parent company are $\rho(A^P, L^P) = \rho(A^S, L^S) = 0.2$ and $\rho(A^P, L^S) = \rho(A^S, L^P) = 0$.[9] The correlation between the assets of (P) and (S), as well as the correlation between their liabilities, are fixed at the same value $\rho = \rho(A^P, A^S) = \rho(L^P, L^S)$ for sensitivity analyses. In the analysis, we compare results for $\rho = 0$ and $\rho = 0.7$.[10] The riskless rate of return is given by $r = 3.5\%$, and the share of the subsidiary's liabilities ceded

[8] Other assumptions and further robustness checks yield similar effects. The numerical sensitivity analyses only serve to provide a better illustration of the theoretical considerations in the previous section.

[9] See, e.g., Gatzert and Schmeiser (2008a, 2008b) for an application to the case of an insurer.

[10] The correlation between different entities of a conglomerate depends on the portfolio composition of assets and liabilities and will differ depending on whether a conglomerate consisting of banks or of insurance companies is considered and may also vary over time.

to the parent company in the quota share retrocession is $\beta = 5\%$. The analysis is conducted using Monte Carlo simulation with 1,000,000 simulation runs on the basis of the same set of random numbers (see Glasserman 2004).[11]

In a first step, diversification and joint shortfall risk are measured without accounting for the conglomerate discount, i.e., given a fixed capital structure. In a second step, we do account for the conglomerate discount by calibrating the initial equity capital so that both equity holders and debt holders receive a net present value of zero (fair condition for both stakeholders). Based on the adjusted values, diversification benefits are compared for the different conglomerate structures. First, we take the individual-firm perspective and assess the solvency situation for each entity (P) and (S). Second, the group-management perspective is taken by calculating the diversification benefit for the whole group, as well as joint shortfall probabilities of the entities.

5.1 Measuring diversification benefits without accounting for the conglomerate discount

For the given initial payment of the debt holders ($D_0 = 99.9$) of firms (P) and (S) and with the same input parameters, the fair initial equity E_0 is 30.1 for both entities.

Individual-company perspective Based on the given capital structure ($D_0 = 99.9$, $E_0 = 30.1$), solvency capital and shortfall probabilities are derived for the individual firms that are included in the different conglomerate structures. Results are displayed in Table 2. Since equity capital is fixed in all cases, the situation is fair only in the solo case, and thus also in the holding case (see (1)), and is not fair for the other conglomerate structures. The left (right) columns in Table 2 show outcomes for $\rho = \rho(A^P, A^S) = \rho(L^P, L^S) = 0(0.7)$.

We first focus on the case where $\rho = 0$ (left column in Table 2), i.e., the cash flows of both companies are uncorrelated. In the holding company case, the solvency capital requirements (Panel A) are the same for both firms (P) and (S) due to the same input parameters. In the parent-subsidiary group, the parent's capital requirements are substantially reduced to 14.9 compared to the holding company case of 29.5, while the subsidiary's *SC* remains stable. This illustrates the group diversification effect, which arises because assets and liabilities of parent and subsidiary are not fully correlated. The introduction of a guarantee or quota share retrocession leads to a slight increase in the parent's *SC* to 15.1 and 17.0, respectively, and to a decrease in the subsidiary's *SC* to 28.0 and 24.1 (from 29.5). Here, the subsidiary benefits from down-streaming diversification. In the integrated model, only one result is shown as the two firms are fully merged into a single entity. Thus, the solvency capital can be shown only for the conglomerate as a whole.

Panel B of Table 2 shows the shortfall probability *SP* for the two firms. In the parent-subsidiary model without CRTIs, the parent's shortfall probability is reduced to near zero. The subsidiary's *SP*, on the other hand, is unaffected by the ownership relation since surplus transfers to the parent occur only in states of solvency.

[11] Some of the calculations are performed using closed-form solutions as laid out in the model framework.

Table 2 The individual-firm perspective for a fixed capital structure, $\rho = \rho(A^P, A^S) = \rho(L^P, L^S)$

	$\rho = 0$		$\rho = 0.7$	
	Parent (P)	Subsidiary (S)	Parent (P)	Subsidiary (S)
Panel A: Solvency capital				
Holding	29.5	29.5	29.5	29.5
Parent-subsidiary	14.9	29.5	29.0	29.5
Parent-subsidiary with guarantee	15.1	28.0	29.0	29.4
Parent-subsidiary with retrocession	17.0	24.1	29.0	28.9
Integrated conglomerate	36.3		56.5	
Panel B: Shortfall probability SP				
Holding	0.34%	0.34%	0.34%	0.34%
Parent-subsidiary	0.02%	0.34%	0.32%	0.34%
Parent-subsidiary with guarantee	0.02%	0.02%	0.32%	0.32%
Parent-subsidiary with retrocession	0.02%	0.10%	0.32%	0.32%
Integrated conglomerate	0.01%		0.25%	

The implementation of guarantees or retrocession, however, leads to a considerable reduction in the subsidiary's shortfall probability, from 0.34% to 0.02% and 0.10%. The extent of the reduction depends on the type of risk transfer. In this case, the parent's shortfall probability does not change, since it makes the CRTI payment only when it is financially able to do so. Thus, the parent's debtholders are not in a worse position when CRTIs are in place, whereas the subsidiary's debtholders benefit. The integrated model has a shortfall probability close to zero.

An increase in the correlation coefficient to $\rho = 0.7$ (right column in Table 2) greatly reduces diversification effects compared to the case without correlation. Hence, a low correlation between the cash flows of the entities is crucial in order to benefit from conglomeration in terms of increased solvency (see also Gatzert et al. 2008).

Group perspective In a second step, we take the group-management perspective and derive relative diversification benefits and joint default probabilities (see Table 3).

The relative diversification benefit in the left column of Table 3 illustrates that in our example, the level of group diversification increases with increasing capital linkage between the entities in the conglomerate. Parent-subsidiary models can increase the diversification benefit by implementing CRTIs. The integrated model has the highest diversification benefit at 38.32%. This result, however, depends on the choice of input parameters. Further analyses reveal that, e.g., a change in the volatility of the subsidiary's liabilities to $\sigma_L^S = 0.2$ and an overall decrease in the safety level to $\Pi^{DPO} = 0.3$ can lead to a higher diversification coefficient for the parent-subsidiary model (without CRTIs) than for the integrated model.

In any case, the relative diversification level is substantially reduced when the assets and liabilities of the two entities are highly correlated ($\rho = 0.7$). Furthermore, for zero correlation, the probability that both entities default at the same time (P_2) is near

Table 3 Group perspective for fixed capital structure in Table 2, $\rho = \rho(A^P, A^S) = \rho(L^P, L^S)$

	Relative diversification benefit		Joint default probability (exactly one entity) P_1		Joint default probability (exactly two entities) P_2	
	$\rho = 0$	$\rho = 0.7$	$\rho = 0$	$\rho = 0.7$	$\rho = 0$	$\rho = 0.7$
Holding	0%	0%	0.68%	0.38%	0.00%	0.15%
Parent-subsidiary	24.58%	0.80%	0.36%	0.36%	0.00%	0.15%
Parent-subsidiary with guarantee	26.68%	0.90%	0.04%	0.34%	0.00%	0.15%
Parent-subsidiary with retrocession	30.07%	1.64%	0.12%	0.34%	0.00%	0.15%
Integrated conglomerate	38.32%	4.23%			0.01%	0.25%

zero for all conglomerate structures and the probability that exactly one of the two firms defaults is lowest for the parent-subsidiary model with guarantee ($P_1 = 0.04\%$). The picture changes tremendously for a correlation coefficient of 0.7, in which case the joint shortfall probabilities are very similar for all models, except the integrated one. In that model, one needs to consider that $SP = P_2 = \mathbb{P}(RBC_1^{\text{int}} < 0)$, i.e., the joint shortfall probability corresponds to the individual one, and hence P_1 is not defined.

5.2 Measuring diversification benefits by accounting for the conglomerate discount

Despite the fact that the analysis in the previous subsection allowed a high degree of comparability because of the fixed capital structure and fixed input parameters, the given capital structure, in general, is no longer fair (in the sense of (1)) when a financial conglomerate is formed due to the conglomerate discount. Specifically, the value of the equity holders' payoff is less than their initial contribution. To obtain a fair situation for all conglomerate structures, we calibrate the equity holders' fair initial payment so that it is equal to the value of their payoff, leaving everything else constant. The fair equity capital values are summarized in Table 4 for $\rho = 0$ (left column) and $\rho = 0.7$ (right column).

As described in the model section of this paper, for $\rho = 0$ the value of the equity holders' payoff in the parent-subsidiary model is reduced one-third, to 19.5, by the diversification effect due to participation in the subsidiary. Implementation of guarantee and retrocession does not influence the fair capital structure as they are settled separately. In particular, accounting for the conglomerate discount by using the fair equity capital in Table 4 ensures the fixed safety level $\Pi_0^{DPO} = 0.1$ for a given payment of $D_0 = 99.9$ by the firms' debt holders. Capital and risk transfer instruments further increase the safety level. In the integrated conglomerate model, the amount of equity capital from (S) can be substantially reduced compared to that required in the parent-subsidiary model. For a correlation coefficient of $\rho = 0.7$, the conglomer-

Table 4 Measuring the conglomerate discount: fair capital structure for different types of financial conglomerates (fair equity capital), $\rho = \rho(A^P, A^S) = \rho(L^P, L^S)$

	$\rho = 0$		$\rho = 0.7$	
	Parent (P)	Subsidiary (S)	Parent (P)	Subsidiary (S)
Holding	30.1	30.1	30.1	30.1
Parent-subsidiary	19.5	30.1	30.1	30.1
Parent-subsidiary with guarantee	19.5	30.1	30.1	30.1
Parent-subsidiary with retrocession	19.5	30.1	30.1	30.1
Integrated conglomerate	30.1	8.0	30.1	27.5

Table 5 Individual-firm perspective for fair capital structure in Table 4 (accounting for the conglomerate discount), $\rho = \rho(A^P, A^S) = \rho(L^P, L^S)$

	$\rho = 0$		$\rho = 0.7$	
	Parent (P)	Subsidiary (S)	Parent (P)	Subsidiary (S)
Panel A: Solvency capital				
Holding	29.5	29.5	29.5	29.5
Parent-subsidiary	24.6	29.5	29.1	29.5
Parent-subsidiary with guarantee	24.6	28.2	29.1	29.4
Parent-subsidiary with retrocession	23.5	25.0	29.1	28.9
Integrated conglomerate	34.8		56.2	
Panel B: Shortfall probability SP				
Holding	0.34%	0.34%	0.34%	0.34%
Parent-subsidiary	0.16%	0.34%	0.32%	0.34%
Parent-subsidiary with guarantee	0.16%	0.07%	0.32%	0.32%
Parent-subsidiary with retrocession	0.16%	0.13%	0.32%	0.32%
Integrated conglomerate	0.01%		0.25%	

ate discount on equity capital nearly disappears as diversification effects tend toward zero. In the integrated model, however, there is a small reduction in equity capital.

Individual-company perspective Based on the fair equity capital values for the different conglomerate structures in Table 4, we next calculate the corresponding solvency capital requirements and shortfall probability for the individual companies (see Table 5), thus explicitly considering the conglomerate discount. The results are then compared to the results of the previous subsection where the conglomerate discount was not taken into consideration, i.e., the capital structure remained unchanged (Tables 2 and 3).

Since the fair equity capital is nearly unchanged for $\rho = 0.7$ compared to the solo case, the results based on fair capital structure are not very different from the results based on the fixed capital structure in Table 2. We thus focus on $\rho = 0$ (left columns in

Table 6 Group perspective for fair capital structure in Table 4 (accounting for the conglomerate discount), $\rho = \rho(A^P, A^S) = \rho(L^P, L^S)$

	Relative diversification benefit		Joint default probability (exactly one entity) P_1		Joint default probability (exactly two entities) P_2	
	$\rho = 0$	$\rho = 0.7$	$\rho = 0$	$\rho = 0.7$	$\rho = 0$	$\rho = 0.7$
Holding	0%	0%	0%	0%	0.00%	0.15%
Parent-subsidiary	8.10%	0.67%	0.68%	0.38%	0.01%	0.16%
Parent-subsidiary with guarantee	10.21%	0.77%	0.48%	0.36%	0.01%	0.16%
Parent-subsidiary with retrocession	17.56%	1.51%	0.21%	0.34%	0.01%	0.16%
Integrated conglomerate	40.85%	4.70%	0.27%	0.34%	0.01%	0.25%

Tables 2 and 5) and find that the group diversification effects for the parent company are substantially reduced when accounting for the conglomerate discount. The reduction in solvency capital, for instance, is much less distinct in the parent-subsidiary model (with and without guarantees or retrocession): the solvency capital in Table 5 decreases from 29.5 to 24.6 instead of from 29.5 to 14.9 when the capital structure is not adjusted accordingly (see Table 2). Similarly, the shortfall probabilities *SP* are much higher in the parent-subsidiary models—especially for the parent company (0.16% vs. 0.02%). In contrast, the integrated model continues to have a very low shortfall risk, similar to the case of fixed capital structure.

Group perspective From the group-management perspective (Table 6), the differences between fair and fixed capital structure are most clear when considering the relative diversification benefit in Tables 3 and 6. In the parent-subsidiary construct, the benefit is reduced from 24.58% to 8.10%; when including a quota share retrocession, the benefit decreases from 30.07% to 17.56%. This is caused by two effects: (1) the available capital of both companies is reduced because of the adjustment of equity capital, and (2) the much higher solvency capital requirements intensify the effect. The probability that exactly one of the two entities in the parent-subsidiary model defaults increases as well, as indicated by the results for the individual shortfall risk.

In contrast to the lower diversification benefit in the parent-subsidiary model, the integrated model shows a 2.5 percentage points higher benefit for $\rho = 0$ given the fair capital structure, even though the available capital is reduced by adjusting E_0^S.

6 Relating numerical and empirical findings

In this section, we relate our theoretical and numerical findings to recent empirical research that investigates the effect of corporate diversification of equity and debt value and potential wealth transfers between equity and debtholders.

Grass (2009), Ammann et al. (2009), and Glaser and Mueller (2010) find that the conglomerate discount exists and that it is significant, even if the market value of debt is used, instead of the book value of debt, when calculating firm value. This is in contrast to the results in Mansi and Reeb (2002), which, according to the above-cited articles, may be due to a sample selection bias or lack of statistical power. Furthermore, Ammann et al. (2009) show that the organizational structure only affects debt value when the degree of diversification increases or decreases and that these effects prevail for only a limited time period, as all subsequent bond issues will again be at par. Thus, there is no considerable increase in the market value of debt compared to the book value, and hence the authors conclude that the risk-shifting argument explains only a limited part of the conglomerate discount. A similar observation is made by Grass (2009), who rejects the hypothesis that mere financial diversification justifies a significant discount on the equity value of conglomerates.

In this paper, we only consider market values for debt and equity and do not explicitly assume that there is a risk shifting or value transfer from shareholders to bond holders, which is in line with the empirical literature. Instead, we assume that after a change in the degree diversification (in the sense of different organizational structures of the conglomerate or an inclusion of capital and risk transfer instruments, leaving everything else constant), equity is immediately withdrawn without including a time lag in such a way that the debt holder value remains unchanged.[12] In particular, the safety level in terms of the default put option value is the same and the total firm value, consisting of market value of equity and debt decreases, which is generally in line with the empirical findings previously mentioned.

The existence and modeling of a conglomerate discount in the first place is consistent with the empirical findings in, e.g., Ammann et al. (2009), and Glaser and Mueller (2010), as well as those in Schmid and Walter (2009), who find significant reductions in firm value for conglomerates. In our model, this conglomerate discount is driven by imperfect correlations between assets and liabilities within and across entities, and by the existence of capital and risk transfer instruments. This can be related to financial diversification in financial conglomerates as demonstrated by Schmid and Walter (2009) and Van Lelyveld and Knot (2009) for the case of the financial services industry in the United States and for bank-insurance conglomerates in the European Union, respectively. Grass (2009), however, empirically shows that the equity call option value may not be significantly affected by a risk reduction of assets in the case of nonfinancial firms and finds that the reduction, on average, amounts to only 0.9%.

At first glance, this particular point is not in line with our numerical examples or our theoretical model; however, there are several arguments that can be made as to why, in principle, our results are consistent with the mentioned empirical findings. First, in contrast to Grass (2009), our theoretical model not only includes correlations between different entities' assets, but also between assets and liabilities within and across entities, effects that are not focused on in Grass (2009) and that may explain

[12]Measuring the conglomerate discount by means of firm value accounts for both shareholder and debt holder value. In this paper, we fixed the debt holders' initial payment and calculated the amount of equity capital needed to reach a fair situation for both debt holders and equity holders, meaning that the market value corresponds to the respective contributions.

a further discount. The model is thus more complex and involves more risks on the liability side, which is especially relevant for bank-insurance conglomerates. In addition and as an extension to the empirical literature, our theoretical setting permits explicitly including capital and risk transfer instruments and clearly differentiating between different types of conglomerates (parent-subsidiary, holding, integrated). For instance, we explicitly take into account that the market value of the subsidiary is an asset for the parent company, and thus consider the specific diversification effects in regard to risk and complexity. Second, the implied correlations between the segments in the Grass (2009) paper have a mean of 0.621 and a median of 0.6. For these correlation values, the conglomerate discount in our model is considerably reduced as well, as can be seen in the tables for $\rho = 0.7$, and so is the diversification benefit with regard to the solvency capital requirements. Third, positive synergy effects, such as reduced costs or lower liabilities, are not considered, which may also generally alleviate reduction of the equity option value. Overall, our model and the numerical results are generally consistent with the empirical results in Grass (2009), and are especially consistent with the empirical findings in Van Lelyveld and Knot (2009) for European bank-insurance conglomerates, seeing as those authors find that the diversification discount varies significantly depending on size, complexity, and risk. This congruency emphasizes our first point with regard to the complexity and risks, as we show that the conglomerate discount, as well as the diversification benefit, strongly depends on the type of conglomerate and the inclusion of capital and risk transfers between entities.[13]

Moreover, compared to empirical literature focusing on the important issue of reasons for the conglomerate discount, we go one step further and show that when accounting for the empirically found conglomerate discount, regulatory diversification benefits and the overall risk situation of each entity involved, measured using the shortfall probability, may not be as comprehensive as it first appears. Specifically, we demonstrate that risk effects with respect to a reduced shortfall probability (not asset risk) and solvency capital requirements are complex and differ for each entity involved, especially in the parent-subsidiary case that is often present in bank-insurance conglomerates, which again is consistent with the findings of Van Lelyveld and Knot (2009).

7 Conclusion

This paper contributes to the literature by providing a new perspective on the risk situation of financial conglomerates. This is accomplished by analyzing diversification effects in a competitive setting, i.e., by accounting for the conglomerate discount in a holding company, a parent-subsidiary group, and in an integrated model using risk-neutral valuation. In addition, we consider capital and risk transfer instruments in the parent-subsidiary group. For both group and individual company management, our model allows studying the impact of diversification on the risk and return situation

[13] In the case of a parent-subsidiary structure, for instance, the conglomerate discount is experienced by only the parent company, and not by the subsidiary.

of financial conglomerates based on solvency capital requirements and shortfall risk and increases transparency in enterprise risk management processes. The results add to the current discussion on group solvency capital requirements in the insurance and banking industry.

In a first step, we show that the choice of a conglomerate structure has a substantial influence on solvency capital requirements. In general, the group solvency capital requirements decrease substantially with the level of integration. However, this effect is alleviated when the entities' cash flows are highly correlated. Capital and risk transfer instruments lead to an increase in solvency capital requirements for the parent and to a decrease in those applicable to the subsidiary. From a regulatory perspective, it is thus important to consider the specific characteristics of the conglomerate when calculating capital requirements, including the degree of participation of each entity, as well as capital and risk transfers between the entities.

In the second step, a meaningful comparison of diversification and insolvency risk is achieved by adjusting the initial equity and debt capital; in other words, we set up a competitive situation. Aside from the solvency situation, the returns to a conglomerate's stakeholders also depend on the type of conglomerate. In particular, diversification reduces shareholder value, which requires a decrease in the initial equity capital (conglomerate discount).

We also investigate shortfall probabilities for the conglomerate and its legal entities. In the parent-subsidiary model, the parent's shortfall probabilities are considerably reduced compared to the solo case, whereas the subsidiary's shortfall risk remains unchanged. Capital and risk transfer instruments from parent to subsidiary do not affect the parent's insolvency risk, but reduce the subsidiary's shortfall risk. Thus, policyholders of both companies profit from this ownership structure in terms of reduced insolvency risk. However, diversification benefits are much lower when the conglomerate discount effect is taken into consideration and, hence, the stakeholders receive risk-adequate returns for their initial contributions. In this respect, our results relativize previous contributions on diversification benefits in financial conglomerates.

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